

ON USING GENERALIZED STOCHASTIC PETRI NETS FOR PERFORMANCE ANALYSIS OF A METROPOLITAN BRT LINE

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ABSTRACT

This paper addresses the problem of assessing quality measures of a public transportation line, including factors such as reliability, commercial speed, and ride comfort. Many existing mathematical models are able to simulate the behavior of a transportation system in time and space. However, a number of these models can be overly expensive and difficult to deal with. A simulation model based on generalized stochastic Petri nets (GSPN) was developed and applied to a BRT line in Recife, Brazil. The model yielded an explanatory power greater than 95%, and the difference between data obtained from a field survey and the results generated by the simulation of the model was statistically insignificant, with a value of 1%. Based on this, we verified that the model is a useful tool for testing potential modifications to a system by means of simulation, because it allows different operational scenarios to be quickly assessed. The developed model also presents several advantages by combining methods of Operational Research (such as queuing theory and graph theory), and adequately represents the stochastic characteristics of the system. Moreover, the GSPN model is simple and practical to modify and implement. Furthermore, the model does not require deep knowledge of mathematical theories, making it extremely useful for specialists of different areas, such as modelers and transport operators or transport authorities, to communicate and trade information.

Key-words: Public Transport; Quality assessment; Mathematical modeling; Simulation; Generalized stochastic Petri nets.

1. INTRODUCTION

Because of growing traffic congestion in urban areas, the current challenge in mobility planning is persuading automobile users to transfer to public transport or non-motorized modes of transport. Therefore, it is fundamental to apply well-performing, high-quality analytical methods to optimize public transport network operations to make it a more attractive alternative. The spatial coverage of roads in most urban areas is extensive, and most cities having urban transportation networks operate bus services. In general, the quality of

these services depends heavily on their degree of priority on the streets (Badia et al, 2016; Rodrigue et al, 2013; Vuchic, 2007).

To be (or to remain) attractive, the performance of public transportation must focus on user needs. With this in mind, many studies have increasingly focused on assessing the factors that influence user perception of the quality of public transportation. The factors valued by users vary based on the particular characteristics of each locality. Nevertheless, some factors are fundamental. Regarding transport by bus, studies suggest that service reliability is one of the most influential factors (Islam et al, 2016; De Onã et al, 2016; Barabino; Deiana, 2013; Redman et al, 2013; Guirao et al, 2016). Reliability refers to how well the schedule can be maintained, which includes: punctuality (compliance with the service timetable), regularity (predictability of travel times), and continuity (constancy of service supply without unexpected interruptions).

Travel times are also among the more valued factors, because a shorter travel time results in more time for other activities and a better quality of life (TRB, 2013). In a wider sense, Tun et al. (2017) consider that operational speed interferes with other fundamental factors such as reliability, supply, and comfort. They understand that with higher speeds, buses may complete more trips, provide a higher frequency of service, and reduce waiting times. Higher frequencies of service enhance the capacity of the transportation system and reduce the number of passengers per trip, thus improving comfort.

Another important factor in determining service quality is the comfort of the users (Islam et al., 2016; Tirachini et al., 2013; Barabino; Di Francesco, 2016; Jain et al., 2014). Aspects related to comfort, such as the number of passengers per vehicle, seat quality, and smoothness of the ride tend to become more relevant as the population income increases (Tirachini et al., 2013). Smoothness is a subjective measure of comfort that can be affected by pavement condition, vehicle condition, and driving style.

The performance of bus public transportation, with regard to reliability and commercial speed, is strongly affected by its degree of priority on urban streets and by the ease of embarkation and disembarkation at stops and terminals. It is also important to guarantee bus priority at crossroads, especially in cases of medium capacity systems such as Light Rail Transit (LRT) and Bus Rapid Transit (BRT) (ITDP, 2016; Hadas; Ceder, 2010).

Mathematical modeling permits predicting the reaction of an existing system to planned interventions with regard to performance and identification of potential

improvements. Using such tools makes it possible to test the impact of proposed operational changes and quantify improvements in service quality (TRB, 2013; Ortúzar; Willumsen, 2011).

In a review, Farias and Borenstein (2014) concluded that operational research has successfully solved a variety of optimization problems in the field of public transportation. Simulation is a frequently used technique in the study of complex systems, such as transportation systems, as it allows reproducing system function in a relatively simple way. Many mathematical models can simulate transportation system behavior in time and space to test alternative plans, although these models can be very expensive and difficult to implement. A simulation model may be executed many times to characterize a specific problem, contribute to identifying potential challenges and to identify potential solutions to address them (Ng et al., 2013; Ortúzar; Willumsen, 2011).

A transportation system may be analyzed as a dynamic discrete event system (DES) composed of a set of interacting elements (passengers, vehicles, physical structures) that shifts a state of events (embarkation and disembarkation, arrivals, and departures), which are discrete features with respect to time. It is possible to find several applications of DES in Petri nets (PN) in fields such as industrial production, computing, and telecommunications. Petri nets permit systems to be governed by a set of equations with a simple graphical representation, allowing system activities to be observed. In general terms, PNs do not require deep knowledge of mathematical theories. They are used extensively to trade information between specialists of different areas, such as modelers and transport operators or transport authorities (Petri, 1966; Murata, 1989; Yen, 2006; Ng et al., 2013).

In urban transportation, PNs have been applied to modeling, performance analysis and control of urban traffic systems, and structural and operational issues of metro transportation lines. The results demonstrate the ability of PNs to offer a graphical representation of nets, and their ability to provide a balance between modeling power and analytical capacity (Tzes et al., 1996; Di Febbraro; Sacco, 2004; Di Febbraro et al., 2016; Dezani, 2012; Ng et al., 2013; Wang et al., 2016; Giglio; Sacco, 2016).

Although performance analysis of public transportation systems using PN has not yet gained considerable traction in the literature, many researchers have concluded that the tool is suitable for the modeling and simulation of urban transportation systems (Giglio; Sacco, 2016; Wang et al., 2016; Yamada et al., 2013; Giua; Seatzu, 2008; Bouyekhf et al., 2003; Di Febbraro et al., 2016; Shi; Huang, 2008; Doloti; Fanti, 2006; Wang et al., 1999).

Abbas-Turki et al. (2002) developed a new class of PN based on Colored Petri nets (CPN) to study the planning of public transport lines in terms of frequency and itineraries. Castelain and Mesghouni (2002) utilized simulations with stochastic timed Petri nets (STPN) to assess the impact of operational planning on the flow of passengers in a multimodal network. This application addresses only the supply of transportation modes on demand by passengers. Kaakai et al. (2007) presented a model based on a hybrid PN to evaluate the performance of components of public transportation stations, such as platforms, stairs, and gates, with regard to avoiding accidents or emergency situations during peak times. Lopez et al. (2011) modeled the BRT Transmilenio of Bogotá in a deterministic timed PN using a multi-agent approach to define the necessary vehicle quantities for attendance peak times.

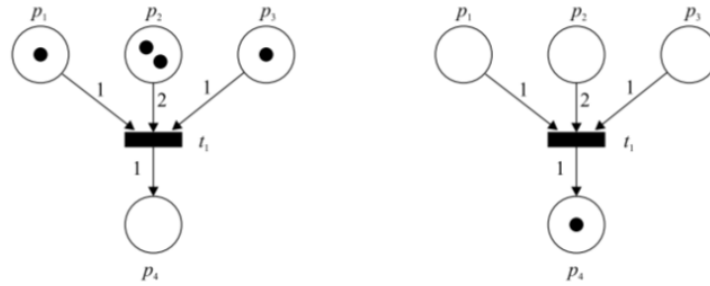
As seen in the literature review, the following advantages of PN in public transportation can be highlighted: i) ease of adjusting networks; ii) use of a simple graphical tool to facilitate simulation of complex systems; iii) versatility for various uses; iv) intuitive and simple modeling process; v) visualization of elements over time to determine the system's behavior; vi) ability to evaluate different situations; vii) use of a widely mathematical tool that includes queuing theory, graph theory and dynamic system simulation.

Due to the potential for PN to be used in assessing the performance of dynamic systems of discrete events, this article aims to develop a generalized stochastic PN model to characterize the performance of a BRT metropolitan line in terms of reliability, comfort and travel time.

2. PETRI NETS: CONCEPTS AND PRACTICE

The Petri net, as proposed by Petri (1966), is a bipartite graph, oriented and valued, whose fundamental idea involves the use of tokens that cross this graph based on events. A PN bipartite graph has two types of node: places and transitions. Places describe possible states or situations of the variable to be modeled. Transitions represent an action executed by the system, or an event. Tokens are points within a place that represent each marking and change places when events occur at the transitions. Each transition is associated with a firing function, which is a rule that defines the time between the transition enabling and the execution of the action or firing of the transition. The standard design of a PN considers that each place is represented by a circle and each transition by a rectangle (see Figure 1).

Figure1 - System status with transition t_1 enabled, left, and after the firing of t_1 , right.



The arcs connect places to transitions, and transitions to places. A weight is associated with each arc, indicating the number of markings required to enable the transition. A transition is enabled when there are sufficient tokens in each place to which it is linked. On the other hand, arcs beginning from a transition indicate the number of tokens generated at a destination place when the transition fires. Thus, the markings circulate toward the arcs while the places are connected to other places via transitions.

There are numerous extensions that add to the basic properties of the PN to model behaviors or specific restrictions that the original pattern does not allow for. These include colored Petri nets (CPN), stochastic timed Petri nets (STPN), and prioritized Petri nets (PPN). The STPN is especially suitable for assessing a system's performance. This assessment allows the performance measures to be extracted as probabilities of certain conditions occurring, as well as the expected number of tokens in a place, the average firing number of a transition over a time, and the average time for a given token to cross a segment of the network (Murata, 1989; Yen, 2006; Kaakai et al, 2007).

Generalized stochastic Petri nets (GSPN), as proposed by Marsan et al. (1984), are an extension of STPN in which stochastic times are mixed to deterministic null times such that both the temporal and the logical evolution of the system can be properly described by the model. The features of a GSPN indicate its compatibility with the study of the performance of a transportation network, since travel time may be modeled as the time that a token requires to cross a network.

Formally, a GSPN model is an 8-tuple: $M_{GSPN} = (P, T, I, O, H, W, PAR, PRED, MP)$, where W is a function defined in the set of transitions. In addition, $M_{\pi} = (P, T, I, O, H, PAR, PRED, MP)$ is a PN model with priorities and inhibiting arcs, called an underlying PN model, where P is a set of places; T is a set of transitions; I , O , and H are functions that describe the input, output, and inhibiting arcs, respectively; PAR is a parameter's set; $PRED$ is a set of

predicates that restricts the parameter's intervals; and MP is the initial marking associated with each place, and which may be a natural number or a parameter that belongs to the set of natural numbers.

GSPN is a continuous-time Markov chain (CTMC). The W function allows the stochastic component of a GSPN model to be defined. This function maps the transitions to positive real functions of GSPN markings. Thus, for any transition, it is necessary to specify a $W(t, M)$ function. For any transition $t_k \in T$, the quantity $W(t_k, M)$ is called the transition rate for a marking M , if t_k is timed, or the transition weight for a marking M if t_k is immediate.

The firing of a transition may signal the end of a time-consuming activity or verify a logical condition. In the first case, timed transitions are used. In the latter, immediate transitions are used. In case of timed transitions, conflict resolution depends on the associated transition times and is obtained by the race rule, which establishes that, when several timed transitions are enabled in a marking M , the transition with the smallest associated time fires first.

Immediate transitions fire when enabled and have priority over timed transitions. However, when two or more immediate transitions are enabled in the same marking, some fixed rule must govern the firing of these transitions. Two types of rules are used in this case. The first is based on a deterministic choice using a priority action. The second rule associates a discrete probability distribution function with a set of conflicting transitions so that conflicts between immediate transitions are randomly solved.

In GSPN, associating random times with a probability density function (PDF) exponential negative to timed transitions ensures that the model's qualitative behavior is the same as that of a PN without a time specification. The use of this PDF ensures that conflicts will be solved without any action, and temporal information provides the metric that allows this solution.

It is possible to define a sequence of timed transitions as a sequence of transitions added to a set of non-decreasing real values that describes the firing time of each transition. This sequence is denoted as $[(\tau(1), T(1)); \dots; (\tau(j), T(j)); \dots]$. The time intervals $[\tau(i), \tau(i + 1)]$ represent the periods for which the timed PN system remains in the marking $M(i)$. This wait time corresponds to a period in which one or more activities is in progress and the status of the system does not change.

The conflict between immediate and timed transitions may cause activities in progress to be preempted when some special event occurs. In the case of timed transitions with an enabling degree greater than one, more attention must be paid to time considerations. From the terminology of queuing theory, the following situations can be considered:

1. *Single-server*: a firing time is defined when the transition is enabled for the first time, and new times are generated after the firing if the transition is still enabled in the new marking. This means that a set of tokens is processed in series, and that the time specification associated with the transition is independent of the enabling degree.
2. *Infinite-server*: each enabled set of tokens is processed as soon as it appears in the transition input. The firing time is generated at this moment and the timers related to all of these enabling sets are executed until zero, in parallel. Several sets of tokens are processed simultaneously. One can say that the global time specifications of the transitions depend directly on the enabling degree.

Presenting these different firing semantics allows defining systems in PN that are graphically simple, without losing any of the characteristics that allow their underlying behaviors to be analyzed. The main reason for introducing temporal specifications in PN models is to test performance indices by simulation. In this approach, from a sequence of tests, it is possible to find all the markings in a network that satisfy a given condition and sum the times spent in the markings of interest. For the purpose of this research, a PN simulation was used to model the performance analysis of a Metropolitan BRT Line in Recife, Brazil.

3. MODEL CONSTRUCTION

Stations and stops are the gateways to public transportation systems where boarding and disembarkation take place. Usually, to reproduce the operation of a public transport line with a GSPN model, the stations and stops are numbered and represented by transitions. In large systems, the number of stops along the corridor can result in a very complex system for this analysis. Thus, to simplify modeling, stations with similar features were grouped as segments. The main concept is that a segment is a station that represents the operational performance of all stations included in that segment.

Data collection aimed to obtain information on the current operation of the aforementioned public transportation line. This information primarily included travel times and the number of passengers boarding and disembarking at each station. Travel time was measured as follows: i) acceleration time (*at*) from the moment the vehicle starts moving until it reaches cruising speed; ii) constant speed time (*cst*) from the moment the vehicle reaches

cruising speed until it starts braking; iii) deceleration time (bt) from the moment the vehicle starts to brake until it completely stops at the station; iv) dwell time (dt), or the time the vehicle remains stopped at the station, which includes door opening and closing as well as embarkation and disembarkation of passengers.

The measurements at each station for a segment are summed for each trip, and the arithmetic mean of these sums is used. For example, let K be a segment composed of two stations E_i and E_j . Assuming that 12 boarding measurements have been taken at each station, the arithmetic mean of these measurements is 50 for E_i and 65 for E_j . Then, to obtain the number of embarking passengers corresponding to segment K , we sum the average number of passengers embarking at each station. This corresponds to the number of passengers boarding at segment K , which is $50 + 65 = 115$.

The software adopted for the construction and simulation of the GSPN model was timeNET from Technische Universität Ilmenau (2015). The input data for the model included the passenger arrival rate, the average length of stays at the stations, the travel time between segments, and the probability of a passenger disembarking along each segment.

- The arrival rate for segment I (λ_i) is the time between passenger arrivals at the point of embarkation, according to Equation (1), where et_i is the average number of passengers embarking per trip on segment i and $headway$ is the time taken for the vehicle to arrive at the segment.

$$\lambda_i = \frac{headway}{et_i} \quad (1)$$

- The dwell time per passenger (dtp_i) is the average time per passenger that the vehicle is stopped at the stations of a segment i . This value is calculated according to Equation (2), where dt_i is the average dwell time in segment i and o_i is the average occupancy of the vehicle upon reaching segment i .

$$dtp_i = \frac{dt_i}{o_i} \quad (2)$$

- The time in motion (tm_{ij}), which is the displacement of the vehicle between two consecutive segments i and j , is as follows:

$$tm_{ij} = at_{ij} + cst_{ij} + bt_{ij} \quad (3)$$

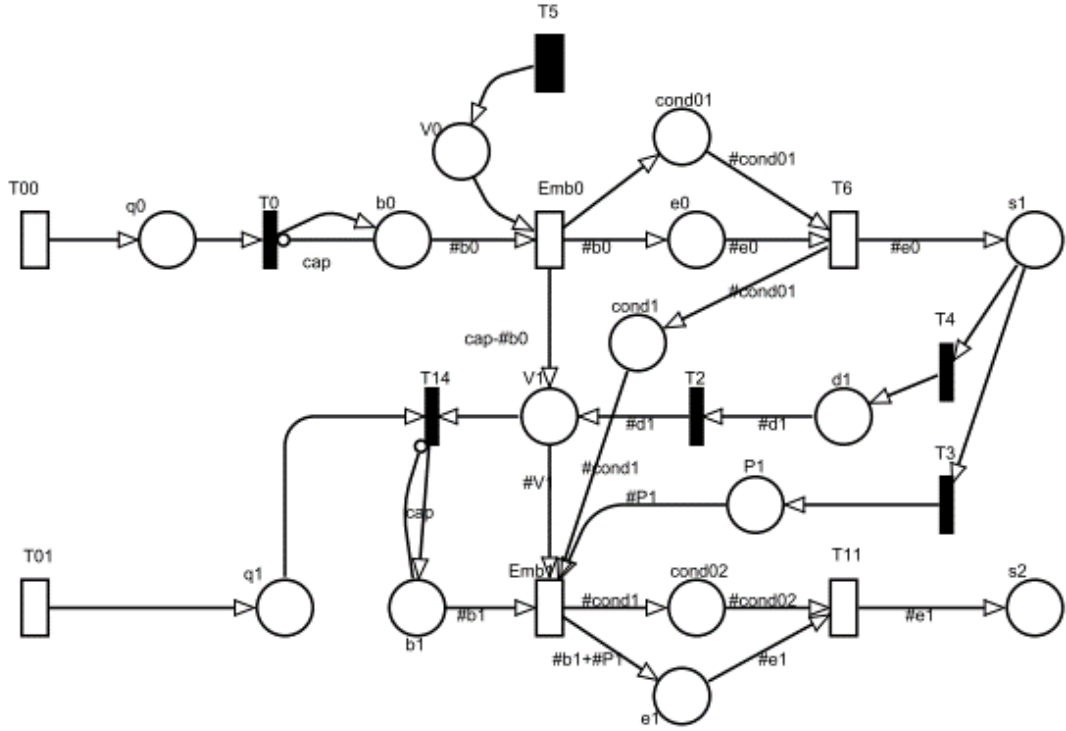
- The probability of disembarking ($P(d)_i$) is given by Equation (4), where d_i is the number of passengers disembarking in segment i .

$$P(d)_i = \frac{d_i}{o_i} \quad (4)$$

Modeling a GSPN presents four basic elements: the places (which describe possible system states or situations), and three types of transitions. Immediate transitions are used to represent events that are important to the system but which do not demand time, such as the vehicle capacity or a passenger's decision whether to disembark at a specific station or to continue the trip. A deterministic transition, whose firing time is determined by the analyst, represents the arrival frequency of a vehicle at the departure station. The exponential transition, whose firing time is described by an exponential probability function, represents the times at which passengers arrive at stations as well as the dwelling times of vehicles at stations and the travel times between consecutive segments. All exponential transitions have a single-server firing semantic.

In the model, the quantity of tokens in a place may symbolize the passengers or vehicles in the system. The tokens in places labeled q_i represent the number of passengers in the boarding queue of segment i that surpass the vehicle passenger capacity and who, therefore, may not board immediately. Tokens in places labeled b_i indicate the number of passengers waiting on the platform at a segment i that actually will embark. Tokens in places labeled e_i and s_i represent the vehicle's occupancy at segment i . Tokens in places labeled d_i , P_i , and V_i represent the number of disembarking passengers, the number that continue the trip, and the number of vacancies in the vehicle when it reaches segment i , respectively. The sum of the tokens in places labeled $cond_{0i}$ and $cond_i$ represents the number of vehicles operating in segment i . Figure 2 illustrates a part of the developed model.

Figure2–Part of the developed GSPN model



Analysis of the permanent behavior of the system was performed using a stationary simulation with a confidence level of 95%, a seed value of 12.345 and at least 50 firings of each transition. The spectral estimation method was used for the confidence interval.

The model predicts, with a confidence interval of 95%, the passenger wait times (wt) at the starting point (S_1), the average number of passengers per vehicle in each segment i (o_i), and the average travel time between segments i and j (t_{ij}). For this, the following equations, based on queuing theory, were incorporated into the model.

$$wt = (q_0 + e_0) \times \lambda_0 \quad (5)$$

$$o_i = \frac{e_i}{cond0_i} \quad (6)$$

$$t_{ij} = (o_i \times dtp_i) + tm_{ij} \quad (7)$$

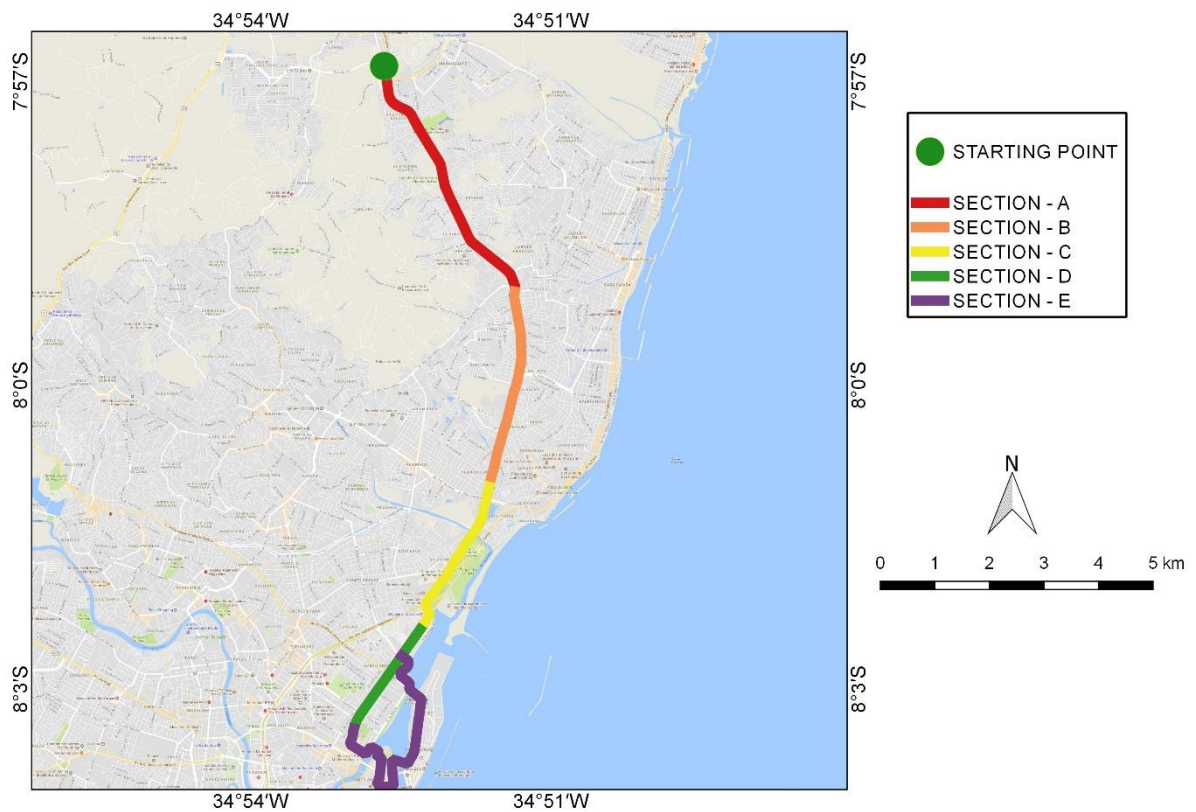
Where q_0 is the average number of tokens in place q_0 , b_0 is the average number of tokens in place b_0 , e_i is the average number of tokens in place e_i for each segment i , and $cond_{0i}$ is the number of vehicles operating in each segment i .

4. MODEL TEST

The developed model was tested on BRT line #1976, which runs through the Metropolitan Area of Recife, Brazil. The North-South Corridor, where the line operates, transports almost 56,000 passengers daily and has a projected goal of transporting 140,000 (ITDP, 2017). This corridor is not completely segregated from the traffic. Hence, its operating problems are correlated with travel times.

According to the metropolitan transport authority (Grande Recife, 2015), BRT line #1976 performs 178 trips per weekday with a headway of six minutes during peak times. The line's path totals 31.85 km over a round trip. In Figure 3, five sections crossed by the BRT in this corridor can be seen. Each of these segments has different characteristics.

Figure3–Sections crossed by BRT line # 1976.



Source: Designed from a Google Earth image (2017).

Section (a), which has a length of 4.80 km and 7 intermediate stations, operates in the central lanes of state highway PE-015 in a segregated manner. Although the corridor is physically segregated from the common traffic lanes, there is some interference in the form of one traffic light for vehicles, 12 u-turns and five pedestrian crossings. Section (b) is located on

the PE-015 highway and is 3.75 km long, of which 3.35 km are physically segregated from the common traffic, and 0.40 km operates in mixed traffic. In this section, there are four intermediate stations, two pedestrian crossings, and three traffic lights.

Section (c), which has a length of 3.20 km and an intermediate station, operates partly on a highway (2.60km) and partly on Cruz Cabugá Avenue (0.60km), which is the main link between the northern metropolitan area and Recife downtown, where the movement of vehicles is quite intense and public transportation is given no special priority. This segment operates entirely in mixed traffic and contains three semaphores. In addition, the intermediate station has a recessed bay, which may hinder the speed with which vehicles can return to the route.

Section (d) operates completely on Cruz Cabugá Avenue and is not separated from common traffic. Along its 2.30 km length, there are three intermediate stations and some interference, such as a pedestrian crossing and 12 traffic lights. The last segment is 5.15 km long and is located in the central area of Recife, where there are no segregated roads or exclusive lanes for public transportation. Section (e) has 6 stations and 11 traffic lights and operates in mixed traffic. Table 1 summarizes the characteristics of each segment of the line.

Table1–Route of BRT line #1976

Impedances/path features	Section					Total
	a	b	c	d	e	
Integrated terminals	1	1	0	0	0	2
Intermediate stops	7	4	2	5	6	24
Returns without traffic light	12	-	-	-	-	12
Pedestrian crossings without traffic light	5	2	-	1	2	10
Traffic lights	1	3	3	12	11	30
Bus-stop bays	-	-	1	-	-	1
Route on state highway (m)	4,800	3,350	2,600	-	-	10,750
Route on Cruz Cabugá Avenue (m)	-	-	600	2,200	-	2,800
Route on Recife downtown (m)	-	-	-	-	5,150	5,150
Route on segregated bus lane (m)	4,800	3,350	-	-	-	8,150
Route on mixed traffic (m)	-	400	3,200	2,200	5,150	5,800
Total segment length (m)	4,800	3,750	3,200	2,200	5,150	19,100
Total length–round trip (m)	9,600	7,500	6,400	3,200	5,150	31,850

For the purpose of analysis, the segments represented in the GSPN model were defined based on the sections in Table 2. The starting point of the line is treated as a separate segment to allow for proper analysis, due to its special features like high demand for embarking and lack of disembarkations.

Table 2 – Segment descriptions

Segment	Section	Direction	Description
1	a	Starting point	From the starting point to the first station of segment 2 (500 m)
2	a	Suburb-city	From the first station of segment 2 to the first station of section b
3	b	Suburb-city	From the first station of section b to the first station of section c
4	c	Suburb-city	From the first station of section c to the first station of section d
5	d	Suburb-city	From the first station of section d to the first station of section e
6	e	City-suburb	From the first station of section e to the first station of section d
7	d	City-suburb	From the first station of section d to the first station of section c
8	c	City-suburb	From the first station of section c to the first station of section b
9	b	City-suburb	From the first station of section b to the first station of section a
10	a	City-suburb	From the first station of section a to the starting point

It is worth remarking that the waiting time at the starting point, the average vehicle occupancy, and the travel time are related to one another, as shown by Equations 5, 6, and 7. The waiting time at the starting point is directly related to the number of passengers arriving at station (e_0) as well as to the average occupancy in segment 1. Travel time depends directly on the average occupancy of the vehicles. Thus, to validate the model, we have compared the average occupancy results for each segment generated by the simulation to data collected in the field. By validating the average occupancy, it can be stated that the model is able to represent the behavior of the three variables of interest.

5. RESULTS AND DISCUSSION

During the field surveys, we obtained information on eight of the eleven trips scheduled between 6 a.m. and 7 a.m. Table 3 shows how the model input data related to the number of passengers per segment were calculated. The arrival rate λ , in seconds, corresponds to the inverse of the quotient of the number of passengers boarding per trip e and the line headway. The probability of landing $p(d)$ on a segment is the average number of landings d divided by the average occupancy of the vehicle in segment o_i .

Table3–Calculation of arrival rate and probability of landing by segment

Segment	e_t	λ (s)	o	d	$p(d)$	$1-p(d)$
1	136.2857	2.64	136.2857	0.0000	0%	100%
2	38.3333	9.39	165.6190	9.0000	7%	93%
3	26.4286	13.62	115.1726	76.8750	46%	54%
4	4.5000	80.00	109.9226	9.7500	8%	92%
5	7.1250	50.53	87.2143	29.8333	27%	73%
6	27.0000	13.33	39.2143	75.0000	86%	14%

7	3.8750	92.90	37.2143	5.8750	15%	85%
8	4.0000	90.00	40.2143	1.0000	3%	97%
9	25.2857	14.24	53.8750	11.6250	29%	71%
10	15.8750	22.68	32.7500	37.0000	69%	31%

The data collected show that the main point of passenger embarkation is the starting station, which has no disembarkation. While most landings take place in segments 3 and 6, segment 3 comprises an important integration terminal in the region where many passengers transfer to complete their travels via other bus lines. Segment 6 runs through the central area of the city, which is the destination of the majority of the passengers of BRT line#1976. In the segments that represent the city-suburb direction of travel, it is observed that the vehicle occupancy is smaller because, in this analysis, we consider only the morning peak period, in which the largest flow travels towards the center of Recife. Table 4 shows the calculation of the model input data related to travel time.

Table4 -Calculation of travel time between segments and dwell time in each segment

Segment	at (s)	cst (s)	bt (s)	tm (s)	dt (s)	o	dtp (s)
1	49.2500	23.1250	10.6250	83.000	109.0000	136.286	0.79979
2	61.2500	268.0000	114.0000	443.250	103.0000	165.619	0.621909
3	71.5000	380.8750	85.7500	538.125	104.1250	115.173	0.904078
4	23.8750	317.0000	29.0000	369.875	27.7500	109.923	0.25245
5	42.8750	418.7500	64.3750	526.000	71.2500	87.214	0.816953
6	70.0000	848.3750	106.8750	1025.250	111.6250	39.214	2.846538
7	16.0000	188.2500	21.8750	226.125	24.3750	37.214	0.65499
8	11.0000	149.0000	21.5000	181.500	20.8750	40.214	0.519094
9	39.5000	298.8750	85.1250	423.500	82.1250	53.875	1.524362
10	57.3750	281.0000	139.7500	478.125	83.7500	32.75	2.557252

It was observed that the longest travel time occurred in segment 6, the central area of Recife and the longest segment of the route at 5.15 km. However, segment 10 is 4.80 km long and has a much shorter travel time than segment 6. This may occur due to the preferential treatment given to public transportation in segment 10, which does not occur in segment 6.

5.1 Model Validation

To test and validate the developed simulation model, we compared the average vehicle occupancy in each segment observed in the field with the results generated by the model simulation, as shown in Table 5.

Table5 -Comparison of the observed values and the values generated by the model

Segment	Vehicle occupancy (l)		Percentage difference
	Observed	Generated by the model	

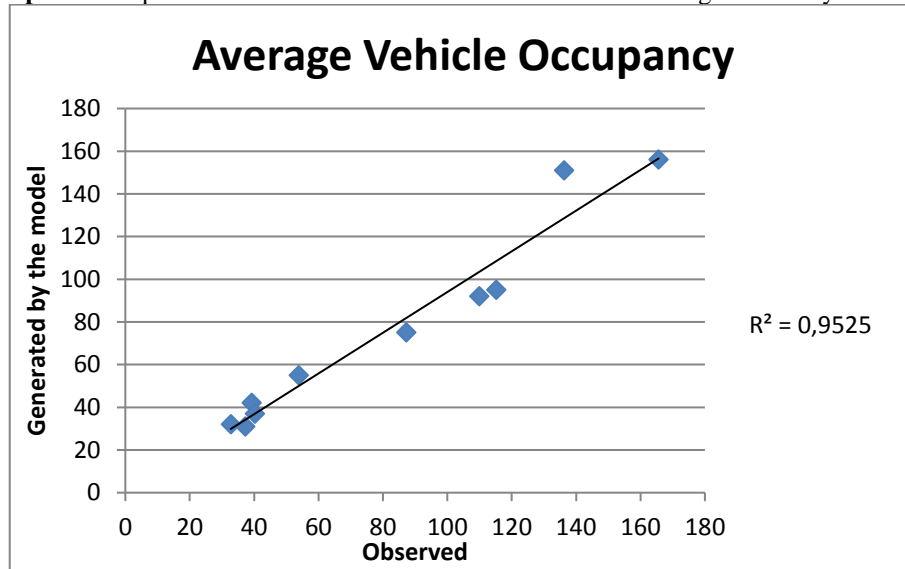
1	136	151	11%
2	166	156	-6%
3	115	95	-18%
4	110	92	-16%
5	87	75	-14%
6	39	42	7%
7	37	31	-17%
8	40	37	-8%
9	54	55	2%
10	33	32	-2%

The largest percentage difference between the result generated in the simulation and the value observed in the field is -18%. Taking into account that the modeled system is highly complex and affected by many variables (the behavior of different individuals, the vehicle and traffic situation, the weather conditions, and the state of physical structures), as Di Febbraro et al. (2016) pointed out, the dynamics of these systems are generally affected by uncertainties. Based on these fundamental principles, the percentage difference measured between the observed and simulated values is considered acceptable.

In addition to the percentage difference, other parameters were examined to validate the model, such as the linear correlation coefficient of the sample (r) and the population (ρ), and the coefficient of determination. The linear correlation coefficient, or Pearson correlation coefficient (r), measures the intensity and direction of the linear relationship of the samples between two quantitative variables. The value of r varies from -1 to $+1$, and the closer the value is to 0 , the lower the degree of linear correlation between the variables. For the occupancy values analyzed, the Pearson coefficient was 0.976 , which demonstrates a strong degree of positive linear correlation between the variables, consistent with what is expected of the model - that is, the real and simulated values vary in the same direction.

The coefficient of determination (R^2) is the quadratic value of the Pearson correlation coefficient and represents the ratio of the variability in the response variable (simulation result) explained by the explanatory variable (real value). This coefficient indicates how well the response variable can be predicted from the explanatory variable. A value of $R^2 = 1$ means that it is possible to predict the answer exactly. In this analysis, $R^2 = 0.9525$, as shown in Graph 1, confirming the high explanatory power of the model. In addition, the trend line, presented in the same graph, indicates the proximity and convergence between the real values and those obtained from the model simulation.

Graph1 - Comparison between the observed values and the values generated by the model



To determine the likelihood of a linear correlation between these variables in the population from which the sample was taken, a hypothesis test was performed with $H_0: \rho = 0$ and $H_1: \rho \neq 0$, where ρ is the population correlation coefficient. For this, we used the Student t distribution with the following test statistic:

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} \quad (8)$$

The calculated t value was 12.671. The critical value, considering a bilateral test with 8 degrees of freedom and a statistical significance of 1% is 3.355. Based on this result, H_0 can be rejected and it can be concluded that the variables possess a linear relationship in the population. Therefore, we can assert that the difference between the values observed and those generated by the model is not significant, and that the model accurately represents the operational performance of the entire line.

The analyzed parameters prove that the developed model is able to represent the average vehicle occupancy accurately, the variables related to waiting time at the starting point, and the travel times. The model is duly validated and effectively able to represent the system behavior and it can be used to assess the performance of a system and the effects of potential sources of interference.

6. CONCLUSIONS

Potential improvements in the quality of a public transportation line may benefit operating companies (by reducing operating costs), users, and society in general. This is

because eventually a good public transportation service may reduce the use of private vehicles, ultimately reducing pollution and congestion. In this context, performance analysis tools for public transportation lines are crucial for identifying critical aspects of the operation and proposing optimization measures.

The variables evaluated in a performance analysis should reflect the quality of the service offered to users, as the users constitute the final consumers whose satisfaction should be the focus of service delivery activities, such as public transportation. Therefore, the optimization of a public transportation service must analyze the current system, evaluate the effects of potential interventions, and suggest actions aimed at improving the service.

There are many models that can simulate the behavior of a transportation system and predict its response to planned interventions. Despite this, studies that help enrich the range of mathematical simulation models available are fundamental, as the most common models currently in use for transportation planning can be expensive and difficult to implement. Hence, it is important to increase the interaction of transport research with other areas, such as operational research and information science.

Therefore, this paper aimed to propose a Petri net model to analyze the performance of public transportation lines with regard to service reliability, comfort, and travel time, which are the aspects of public transportation most valued by users. The intent is for this tool to contribute to the planning of transportation networks based on its assessment of the positive and negative aspects of operation, as well as evaluation of the effects of potential interventions on the quality of the service.

Among the available types of Petri nets, a generalized stochastic Petri net (GSPN) was chosen, because it is a comprehensive instrument that incorporates several methods of operational research, such as queuing theory and graph theory. Moreover, it adequately represents the characteristic stochastic system, and remains a simple and practical model to modify and work with. In addition, the model developed in this paper has several advantages, as it makes it easy to adjust networks, is a simple graphical tool, utilizes an intuitive and simple modeling process, and allows visualization of elements over time. These visual characteristics have the potential to facilitate understanding between modelers and transport managers.

To validate the model, it was applied to a BRT line in the Metropolitan Region of Recife, after which a comparison was made between average vehicle occupancy data obtained

from a field survey and the results generated by the simulation of the model. The BRT line#1976 was selected because it is an extremely important line that serves the main trip generator hub of the metropolitan area, the center of Recife, which is home to a variety of industries and activities, including commerce, education, and health. This area generates jobs and attracts the population of the entire Metropolitan Region of Recife.

The GSPN simulation model exhibited an explanatory power greater than 95%, which means that it was able to accurately represent the behavior of the system with regard to waiting times at the starting point (an indicator of the reliability of the line), the average vehicle occupancy (an indicator of user comfort), and travel times (which are an indicator of the average speed of the line). A difference of 1% was found between the data obtained the field survey and the results generated by the simulation of the model, which was not statistically significant.

By validating the results, we verified that the elaborate model is a useful tool for testing, by means of simulation, potential modifications to the system, as it allows different operational scenarios to be quickly assessed. Tests that would be impractical in a real system can be extensively repeated for a variety of configurations with the aid of this model. The model presents near-real performance and is easy to use for experimentation. Therefore, it is adequate and feasible for analyzing the performance of public transportation lines.

The use of mathematical modeling and simulation tools for analyzing system performance is a fundamental area of transportation research, since it allows both quantitative and qualitative analysis of the system, as well as verification of the behavior of the system in response to operational or physical modifications. Generalized stochastic Petri nets can add to the study of transportation using simple and easily understood formal mathematical modeling.

The model developed is limited to assessing a public transportation line in terms of its reliability, average operational speed, and comfort, which it accomplishes by analyzing service frequency, embarkation and disembarkation, degree of segregation of the bus corridor from general traffic, and priority level during signaling intersections. However, there are several other aspects that could be better evaluated in future studies. Further research may add other metrics of service quality to the analysis. Additionally, a potential research niche involves the characterization of a Petri net model that allows the operational performance of public transportation integration terminals to be analyzed.

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